THERMAL NOISE MEASUREMENTS

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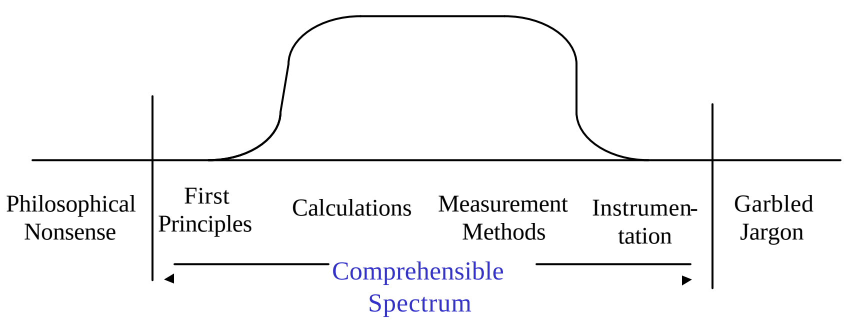
INTRODUCTION

- General Content:



- Outline
  - – Basics
    - Nyquist, Quantum effects, limits
    - Noise Temperature Definition
    - Microwave Networks & Noise
  - – Noise-Temperature Measurement
    - Total-power radiometer
      - – general
      - – simple, idealized case
      - – not so simple case
    - Uncertainties
      - – simple case
      - – not so simple case—not today
    - Adapters

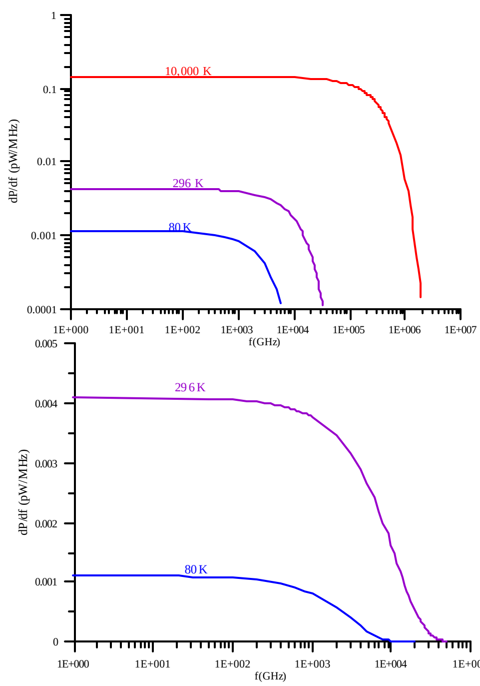
- Outline (cont'd)
  - – Noise Figure & Parameters
    - Noise Figure defined
    - Simple, idealized NF measurement
    - Noise parameters
    - Wave representation of noise matrix
    - Measuring noise parameters
    - Uncertainties
    - Noise in differential amplifiers
  - – Noise Standards & Sources
    - Not covered here
  - – References

I. BASICS

Nyquist Theorem

- Derivation:
  - – Electr. Eng. [1-4]
  - – Physics, Stat. Mech. [4]
- For passive device, at physical temperature T, with small *Df*,

$$\langle P_{avail}(f)\rangle = \frac{hf}{e^{hf/(k_B T)}-1} Df$$



$$\langle P_{avail}(f)\rangle = \frac{hf}{e^{hf/(k_B T)}-1} Df$$

Note: <u>very</u> small powers.

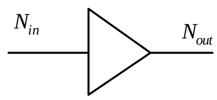
- Limits
  - small f : $<P_{avail}> \approx k_B T\, Df\, [1 - hf/(2k_B T)]$
    $\approx k_B T\, Df$
  - large f : $\rightarrow 0$
  - knee occurs around f(GHz) $\approx$ 20 T(K)
- Quantum effect
  - h/k_B = 0.04799 K/GHz
  - So at 290 K, 1 % effect at 116 GHz
    at 100 K, 1 % effect at 40 GHz
    at 100 K, 0.1 % effect at 4 GHz
    30 K @ 40 GHz $\rightarrow$ 6.4%, 0.26 dB

NOISE TEMPERATURE

- What about active devices? Can we define a noise temperature?
- Several different definitions used:
  - delivered vs. available power
  - with or without quantum effect
    i.e., does $T_{noise} \propto P_{avail}$ ("power" definition), or is T_{noise} the physical temperature that would result in that value of P_{avail} ("equivalent-physical-temperature" definition)?

- IEEE [5]: "(1)(general)(at a pair of terminals and at a specifice frequency) the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual terminals."
  and
  "(4)(at a port and at a selected frequency) A temperature given by the exchangeable noise-power density divided by Boltzmann's constant, at a given port and at a stated frequency."

- We (I) will use second definition, noise temp ≡ available noise-power density divided by Boltzmann's constant.
- It is the common choice in international comparisons [6] and elsewhere [7].
- It is much more convenient for amplifier noise considerations (at least for careful ones)



$N_{out} = GN_{in} + N_{amp}$

If $N = kT$, then $N_{in} = kT_{in}$, etc., and $T_{out} = GT_{in} + GT_e$

But if we use the “equivalent physical temperature” definition, then $N_{in} = \dfrac{hf}{e^{hf/kT_{in}} - 1}$

and similarly for the others, and so $\dfrac{hf}{e^{hf/kT_{out}} - 1} = G\left(\dfrac{hf}{e^{hf/kT_{in}} - 1} + \dfrac{hf}{e^{hf/kT_e} - 1}\right)$.

Solving for T_{out}, we would get

$$T_{out} = \frac{hf}{k}\left\{\ln\left[1 + \frac{1}{G}\left(\frac{1}{\left(e^{hf/kT_{in}} - 1\right)} + \frac{1}{\left(e^{hf/kT_e} - 1\right)}\right)^{-1}\right]\right\}^{-1}$$

- So $P_{avail} = k_B T_{noise} Df$
- And for passive devices,

$$T_{noise} = \frac{1}{k_B}\left[\frac{hf}{e^{hf/(k_B T)} - 1}\right] \approx T_{phys}$$

- Convenient to define “Excess noise ratio”

$$ENR_{(avail)} \equiv \frac{T_{(avail)} - T_0}{T_0} \qquad T_0 = 290\,K$$

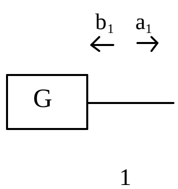
T=9500 K $\Rightarrow$ ENR $\approx$ 15.02 dB

No matter what definition of noise temperature you choose, it is helpful to state your choice.

MICROWAVE NETWORKS & NOISE [8,9]

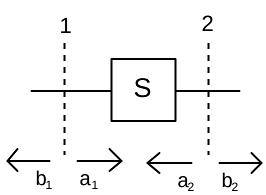
- Assume lossless lines, single mode.
- Travelling-wave amplitudes a, b.
- Normalized such that $P_{del} = |a|^2 - |b|^2$ is spectral power density.
- May be a little careless about B; assume that it's 1Hz where needed.

- Describe (linear) one-ports by



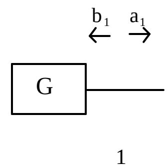
$$a_1 = G_G b_1 + \hat{a}_G$$

- And (linear) two-ports by



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

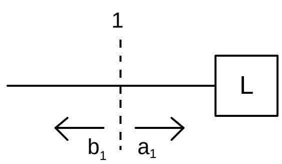
- Available power:



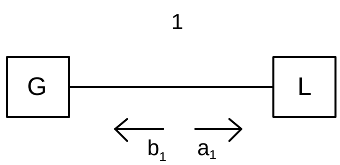
$$P_G^{avail} = \frac{|\hat{a}_G|^2}{1-|G_G|^2}$$

$$\left\langle |\hat{a}_G|^2 \right\rangle = \left(1-|G_G|^2\right) k_B T_G$$

- Delivered power:

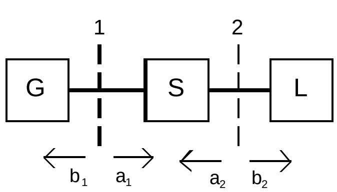


$$P_1^{del} = |a_1|^2 - |b_1|^2 = |a_1|^2 \left(1-|G_L|^2\right)$$



Mismatch Factor

$$M_1 = \frac{P^{del}}{P^{avail}} = \frac{\left(1-|G_L|^2\right)\left(1-|G_G|^2\right)}{|1-G_L G_G|^2}$$



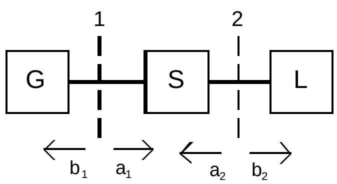
Efficiency

$$h_{21} = \frac{P_2^{del}}{P_1^{del}} = \frac{|S_{21}|^2\left(1-|G_L|^2\right)}{|1-G_L S_{22}|^2\left(1-|G_{SL}|^2\right)}$$

$$= \frac{|S_{21}|^2\left(1-|G_L|^2\right)}{|1-G_L S_{22}|^2 - |(S_{12}S_{21}-S_{11}S_{22})G_L + S_{11}|^2}$$

- Available power ratio:

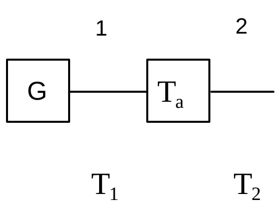
$$a_{21} \equiv p_{2,avail}/p_{1,avail} \;\; (\hat{b}_1\,,\, \hat{b}_2 = 0)$$



$$a_{21} = \frac{|S_{21}|^2\left(1-|G_G|^2\right)}{|1-G_G S_{11}|^2\left(1-|G_{GS}|^2\right)}$$

$$G_{GS} = S_{22} + \frac{S_{12}S_{21}G_G}{1-G_G S_{11}}$$

- Temperature translation through a passive, linear, 2-port (attenuator, adapter, line, ...)



$$P_2^{avail} = a_{21}P_1^{avail} + f_0(T_a)$$
$$T_2 = a_{21}T_1 + f(T_a)$$

Say $T_1 = T_a$, then T_2 must $= T_a$, so

$$T_2 = T_a = a_{21}T_a + f(T_a)$$
$$f(T_a) = (1-a_{21})T_a$$

and therefore

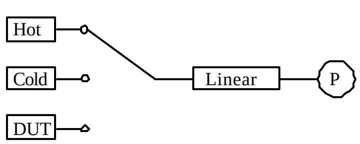
$$T_2 = a_{21}T_1 + (1-a_{21})T_a$$

II. NOISE-TEMPERATURE MEASUREMENT

Total-Power Radiometer [10-12]

- Two principal types of radiometer for noise-temperature measurements are Dicke radiometer and total-power radiometer [10].
- Total-power radiometer is most common for lab use, & that's what we'll discuss.

- Simple case: symmetric, matched (all G's = 0)



Matched $\rightarrow p_{del} = p_{avail}$

Linear $\rightarrow P = a + bp_{del} = a + bp_{avail}$

2 standards (h,c) determine a, b:

$$a = P_c - bk_B BT_c \qquad Bk_B b = \frac{P_h - P_c}{T_h - T_c}$$

So $T_x = T_c + \frac{(Y_x - 1)}{(Y_h - 1)}(T_h - T_c)$, where $Y_x = \frac{P_x}{P_c}$, $Y_h = \frac{P_h}{P_c}$

- Not-so-simple case (unmatched, asymmetric) Three complications:
  - $p_{del} = Mp_{avail}$
  - $p_{del,rad} = h\, p_{del,G}$, and $h_x \neq h_h \neq h_c$
  - $a, b = a(G), b(G)$

  – Handle first two by measuring and correcting.

– For dependence of *a* and *b* on *G*, have three choices:
  - tune so that $G_h = G_c = G_x$ (very narrow frequency range, need special standards)
  - characterize dependence on *G* (broadband, but a lot of work, and difficult to get good accuracy)
  - isolate (easy, accurate, but limits frequency range & difficult at low frequency)

– If isolate, *a* and *b* are (almost) independent of the source, and

$$T_x = T_{amb} + \left(\frac{M_S h_S}{M_x h_x}\right)\frac{(Y_x - 1)}{(Y_S - 1)}(T_S - T_{amb})$$

Uncertainties

- Simple case (matched):

$$T_x = T_a + \frac{(Y_x - 1)}{(Y_S - 1)}(T_S - T_a) \cancel{\frac{M_h h_h}{M_x h_x}}$$

typically around 1 %

about 1 or 2%

small uncert, but linearity concern

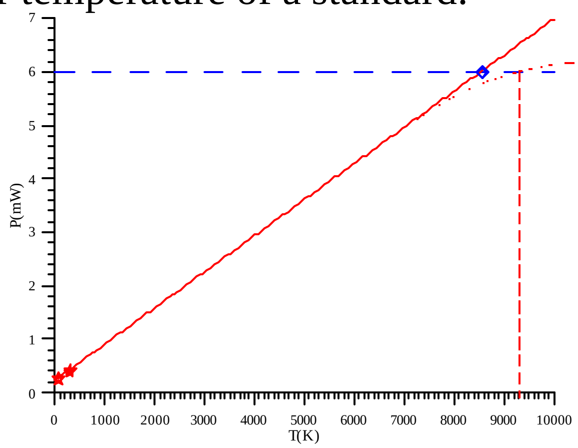
Uncert “should” be negligible

- Simple-case uncerts (cont'd)
  - drift: temperature stability/control important (effect minimized by frequent switching to standards)
  - connector variability: hard to do much better than 0.1%, easy to do considerably worse.
  - *Da*, *Db*: depends on details of system, can make a crude estimate:

$$T_{rev} \sim T_e, \quad |DG| \sim 0.05 \text{ or } 0.1$$

$$\text{So } DT_{in} \sim 0.05 \text{ or } 0.1 \times T_e$$

- linearity: serious concern if T_x very different from standards, less (but some) worry if T_x near temperature of a standard.



- Uncertainties (more careful case)
  (Numbers are for NIST case) [13,14]
  - Radiometer equation:

$$T_x = T_{amb} + \frac{M_S h_S}{M_x h_x} \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_{amb})$$

  - Ambient standard:

$$\frac{u_{T_x}(amb)}{T_x} = \left| \frac{T_x - T_S}{T_a - T_S} \right| \frac{T_a}{T_x} \varepsilon_{T_a}, \quad \varepsilon_{T_a} = \frac{0.1K}{296K} = 0.034\%$$

  - "Other" standard:

$$\frac{u_{T_x}(S)}{T_x} = \left| 1 - \frac{T_a}{T_x} \right| \left| \frac{T_S}{T_a - T_S} \right| \frac{u_{T_S}}{T_S}, \quad \frac{u_{T_S}}{T_S} = 0.2\% (NIST\ W.G.),\ 0.8\% (NIST\ coax)$$

  - Path asymmetry: (zero if connect to same port)

$$\frac{u_{T_x}(h/h)}{T_x} = \left| 1 - \frac{T_a}{T_x} \right| u_{h/h}, \quad u_{h/h} = 0.2\% \ to\ 0.56\%$$

  - Mismatch:

$$\frac{u_{T_x}(M/M)}{T_x} = \left| 1 - \frac{T_a}{T_x} \right| u_{M/M}, \quad u_{M/M} \approx 0.2\%$$

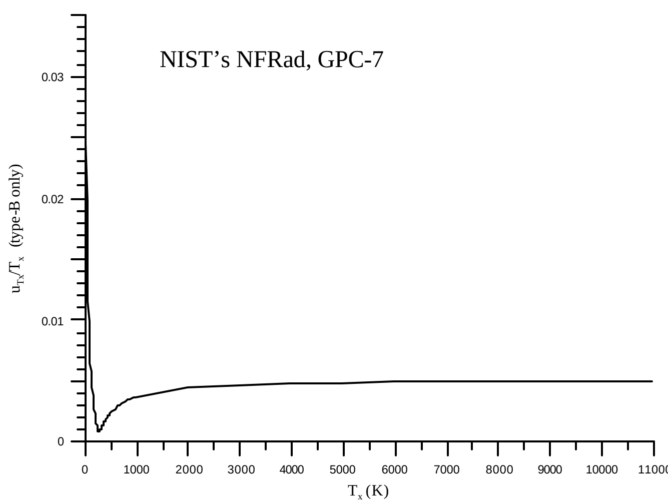
– Connectors:

$$\frac{u_{T_x}(conn)}{T_x} = u_0\left|1-\frac{T_a}{T_x}\right|\sqrt{f(GHz)}, \quad u_0 \approx 0.053\,\%\ to\ 0.069\,\%$$

(depending on connector type)

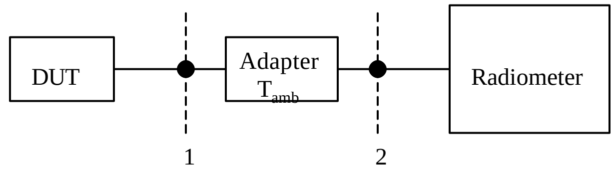
– Other: Nonlinearity, imperfect isolation, power ratio measurement, and broadband mismatch/frequency offset all lead to small (<0.1%) uncertainties for T_x around 10 000 K (for us/NIST).

- u_T(Type-B)/T as a function of T
  Standard relative uncertainty (1σ)



Adapters

- Measure T at 2, want T at 1.



$$T_2 = \alpha_{21} T_{DUT} + (1-\alpha_{21}) T_{amb}$$

So

$$T_{DUT} = \frac{T_2 - (1-\alpha_{21}) T_{amb}}{\alpha_{21}}$$

III. NOISE FIGURE & PARAMETERS

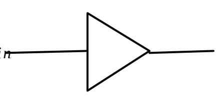
Noise Figure Defined

- Want a measure of how much noise an amplifier adds to a signal or how much it degrades the S/N ratio.

- Define Noise Figure, IEEE [15]:
(at a given frequency) the ratio of total output noise power per unit bandwidth to the portion of the output noise power which is due to the input noise, evaluated for the case where the input noise power is $k_B\, T_0$, where T_0 = 290 K. (vacuum fluctuation comment)
- Noise figure & signal to noise ratio[16]:

$$\frac{(S/N)_{in}}{(S/N)_{out}} = \frac{S_{in}/290K}{GS_{in}/(G\times 290K + N_{amp})} = \frac{G\times 290K + N_{amp}}{G\times 290K} = F$$

- Effective input noise temperature:

S_{in} , N_{in}  $S_{out} = G\, S_{in}$

$$N_{out} = GN_{in} + N_{amp} = Gk_B\boldsymbol{T_{in}} + \boldsymbol{N_{amp}}$$

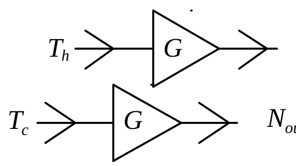
Define $N_{amp} \equiv Gk_BT_e$

So $N_{out} = Gk_B(T_{in} + T_e)$

Noise Figure $F = \frac{Noise\ out}{Noise\ in} = \frac{G(T_0 + T_e)}{GT_0}$ $\qquad F(dB) = 10\log_{10}\left(\frac{T_0 + T_e}{T_0}\right)$

Note: G, F, T_e all depend on G_{source}.

Simple Case, all G 's equal



$N_{out,h} = Gk_B(T_h + T_e)$

$N_{out,c} = Gk_B(T_c + T_e)$

Combine & solve:

$$G = \frac{N_h - N_c}{k_B(T_h - T_c)} \qquad T_e = \frac{N_c T_h - N_h T_c}{N_h - N_c} = \frac{T_h - YT_c}{Y-1} \qquad \text{where } Y = N_h/N_c$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T_h - YT_c}{(Y-1)T_0} = \frac{ENR}{Y-1} + \left(\frac{Y}{Y-1}\right)\left(\frac{T_0 - T_c}{T_0}\right) \approx \frac{ENR_h}{(Y-1)}$$

if $T_c \approx T_0$

$$F\,(dB) \approx ENR_h\,(dB) - (Y-1)(dB)$$

(290 K → ~63 °F)

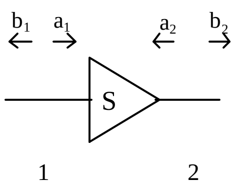
Noise Parameters

- But that's just for one value of G_{source}. Want to determine F or T_e for any G_{source}. So parameterize dependence on G_{source}.
- Several parameterizations in use; most common are variants of the IEEE [17] form. Particular IEEE form we use is [18]

$$T_e = T_{e,\min} + t\frac{|G_G - G_{opt}|^2}{\left(1 - |G_G|^2\right)|1 + G_{opt}|^2} \qquad t = 4\frac{R_n}{Z_0}$$

Wave Representation of Noise Matrix

- For microwave radiometry, wave representation [18-23] provides more flexibility.
- Linear 2-port:



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

- Noise matrix is defined by

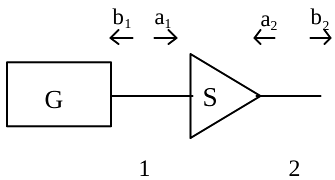
$$N_{ij} = \langle b_i b_j^* \rangle$$

or $\hat{N}_{ij} = \langle \hat{b}_i \hat{b}_j^* \rangle$ for intrinsic noise matrix

- Four real noise parameters:

$$\langle |\hat{b}_1|^2 \rangle, \langle |\hat{b}_2|^2 \rangle, \langle \hat{b}_1 \hat{b}_2^* \rangle$$

- Output noise temperature T_2



$$k_B T_2 = \frac{|S_{21}|^2}{\left(1-|G_{GS}|^2\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$N_G = \frac{\left(1-|G_G|^2\right)}{|1-G_G S_{11}|^2} k_B T_G$$

$$N_1 = \left|\frac{G_G}{1-G_G S_{11}}\right|^2 \left\langle |\hat{b}_1|^2 \right\rangle$$

$$N_2 = \left\langle |\hat{b}_2 / S_{21}|^2 \right\rangle$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{G_G}{(1-G_G S_{11})}\left\langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \right\rangle\right]$$

- So for T_e we have

$$T_e = \frac{|G_G|^2}{(1-|G_G|^2)} X_1 + \frac{|1-G_G S_{11}|^2}{(1-|G_G|^2)} X_2 + \frac{2}{(1-|G_G|^2)} \mathrm{Re}\left[(1-G_G S_{11})^* G_G X_{12}\right]$$

where $k_B X_1 \equiv \left\langle |\hat{b}_1|^2 \right\rangle$, $k_B X_2 \equiv \left\langle |\hat{b}_2 / S_{21}|^2 \right\rangle$, $k_B X_{12} \equiv \left\langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \right\rangle$

- Whereas IEEE parameterization is

$$T_e = T_{e,\min} + t \frac{|G_G - G_{opt}|^2}{\left(1-|G_G|^2\right)|1+G_{opt}|^2}$$

- We can relate the two:

$$t = X_1 + |1+S_{11}|^2 X_2 - 2\,\mathrm{Re}[(1+S_{11})^* X_{12}]$$

$$T_{e,\min} = \frac{X_2 - |G_{opt}|^2 \left[X_1 + |S_{11}|^2 X_2 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)\right]}{\left(1+|G_{opt}|^2\right)}$$

$$G_{opt} = \frac{h}{2}\left(1 - \sqrt{1 - \frac{4}{|h|^2}}\right)$$

where $h = \dfrac{X_2\left(1+|S_{11}|^2\right) + X_1 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)}{(X_2 S_{11} - X_{12}}$

- Going the other way,

$$X_1 = T_{e,\min}(|S_{11}|^2 - 1) + \frac{t\,|1 - S_{11}G_{opt}|^2}{|1+G_{opt}|^2}$$

$$X_2 = T_{e,\min} + \frac{t\,|G_{opt}|^2}{|1+G_{opt}|^2}$$

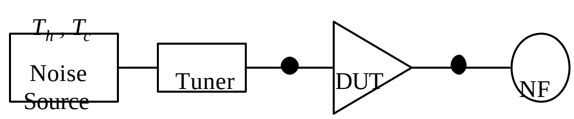
$$X_{12} = S_{11}T_{e,\min} - \frac{tG_{opt}^*(1 - S_{11}G_{opt})}{|1+G_{opt}|^2}$$

note bound implied by $X_1 > 0$.

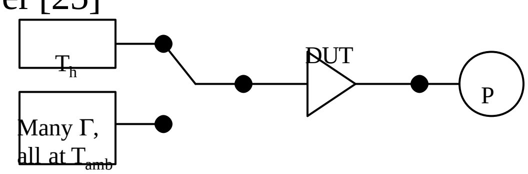
Measuring Noise Parameters

- Many different methods [18,20,22,24-34], most based on IEEE parameterization.
- Basic idea of (almost) all methods is to
  - present amplifier (or device) with a variety of different input terminations (*G* & *T*),
  - have an equation for the "output" in terms of the noise parameters and known quantities (*G* 's, *T*'s, S-parameters),
  - determine noise parameters by a fit to the measured output.
  - Need good distrib. of *G* 's in complex plane.

- "Output" can be
  - Noise figure [24]

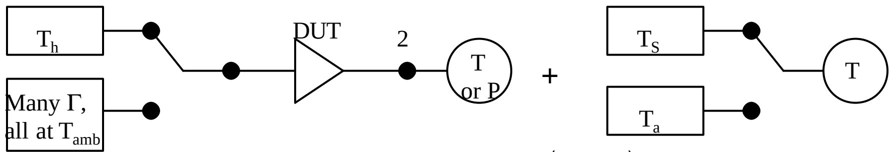


  - Power [25]



  - Note: output *G*, matching, available power, etc.

- Noise-matrix approach [22,23,30] to measuring noise parameters:



$$k_B T_2 = \frac{|S_{21}|^2}{\left(1-|G_{GS}|^2\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$G_{GS} = S_{22} + \frac{G_G S_{12} S_{21}}{(1-G_G S_{11})}$$

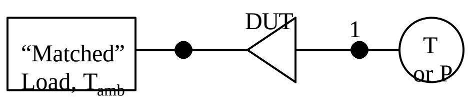
$$N_G = \frac{\left(1-|G_G|^2\right)}{|1-G_G S_{11}|^2} k_B T_G$$

$$N_1 = \left|\frac{G_G}{1-G_G S_{11}}\right|^2 \left\langle |\hat{b}_1|^2 \right\rangle$$

$$N_2 = \left\langle |\hat{b}_2 / S_{21}|^2 \right\rangle$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{G_G}{(1-G_G S_{11})}\left\langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \right\rangle\right]$$

- Supplemental measurement (noise matrix) [27,31] — not today



$$k_B T_1 = \frac{1}{\left(1-|G_{GS}'|\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$G_{GS}' = S_{11} + \frac{G_G S_{12} S_{21}}{(1-G_G S_{22})}$$

$$N_G = \frac{|S_{12}|^2\left(1-|G_G|^2\right)}{|1-G_G S_{22}|^2} k_B T_G$$

$$N_1 = \left\langle |\hat{b}_1|^2 \right\rangle$$

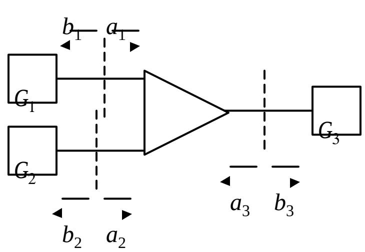
$$N_2 = \left|\frac{S_{12} S_{21} G_G}{1-G_G S_{22}}\right|^2 \left\langle |\hat{b}_2 / S_{21}|^2 \right\rangle$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{S_{12} S_{21} G_G}{(1-G_G S_{22})}\left\langle \hat{b}_1^* (\hat{b}_2 / S_{21}) \right\rangle\right]$$

- Noise-Parameter Uncertainties
  - Monte Carlo method is probably the most practical [26,35-38]
  - Some general approximate features [38]:
    - Uncerts in G and T_{min} (& F_{min}) are dominated by uncert in T_{h}. 0.1 dB uncert in $T_{\mathrm{h}} \rightarrow \sim$ 0.1 dB uncert in G and F_{min}.
    - Uncerts in G_{opt} are dominated by uncerts in G_G's. Uncert in Re or Im G_{opt} is ~ 3 or 4× uncert in Re or Im G_G (for 13 terminations).
    - t is sensitive to just about everything.
    - T_{amb} is not a major factor, because it is known much better than T_{h}. Note, however, that it could affect T_{h} or the amplifier properties.

Noise in Differential Amplifiers

- Simple case, all G's = 0; full treatment in [39].
- Input ports 1 & 2, output port 3. Ideally, $b_3 \propto (a_1 - a_2)$.



$$G_1 = G_2 = G_3 = 0$$

$$a_\pm \equiv \frac{(a_1 \pm a_2)}{\sqrt{2}}$$

$$S_{3\pm} \equiv \frac{(S_{31} \pm S_{32})}{\sqrt{2}}$$

$$b_3 = S_{31}a_1 + S_{32}a_2 + \hat{b}_3$$

$$= S_{3-}a_- + S_{3+}a_+ + \hat{b}_3$$

$$G_{31} = |S_{31}|^2,\ \ G_{32} = |S_{32}|^2,\ \ G_{3-} = |S_{3-}|^2,\ \ G_{3+} = |S_{3+}|^2$$

$$G_{31} + G_{32} = G_{3-} + G_{3+}$$

- Output noise power per unit BW at port 3 is given by $N_3 = \left\langle \left| S_{31} a_1 + S_{32} a_2 + \hat{b}_3 \right|^2 \right\rangle$

- If uncorrelated noise sources T_1 and T_2 are input, then

$$N_3 / k_B = G_{31} T_1 + G_{32} T_2 + \hat{T}_3$$
$$\hat{T}_3 = (G_{31} + G_{32}) T_e = (G_{3-} + G_{3+}) T_e$$

- So to determine T_e and the gains, measure with different T_1 and T_2's.

- Assume a hot and a cold source for each input port: T_{h1}, T_{c1}, T_{h2}, T_{c2}.
- Let $N_{3,\mathrm{hc}}$ be the ouput noise power at port 3 for the hot source on port 1 & the cold source on port 2, etc. Then (ignoring k_B)

$$N_{3,hh} = G_{31} T_{h1} + G_{32} T_{h2} + \hat{T}_3$$
$$N_{3,hc} = G_{31} T_{h1} + G_{32} T_{c2} + \hat{T}_3$$
$$N_{3,ch} = G_{31} T_{c1} + G_{32} T_{h2} + \hat{T}_3$$
$$N_{3,cc} = G_{31} T_{c1} + G_{32} T_{c2} + \hat{T}_3$$

- Four equations, three unknowns. Measure all four & fit, or measure any three & solve.
- So we can determine $\hat{T}_3$, G_{31}, and G_{32}.
- Therefore, we can determine T_e and (G_{3+}+G_{3-}) for differential & common modes from hot-cold measurements with uncorrelated noise sources on the physical ports 1 & 2.

- More work required to get G_{3+} and G_{3-} separately:
  - correlated inputs to ports 1 & 2
  - approximate $G_{3-} >> G_{3+}$, so $G_{3-} \approx G_{3+} + G_{3-}$
  - measure S_{3-} some other way

- Simple example: say you have just one hot source ($T_{h1}=T_{h2}$ and no hh measurement) and cold is just ambient ($T_{c1}=T_{c2}$), then have

$$\hat{T}_3 = \frac{(T_h + T_c)}{(T_h - T_c)} N_{3,cc} - \frac{T_c}{(T_h - T_c)} (N_{3,hc} + N_{3,ch})$$

$$G_{31} = \frac{N_{3,hc} - N_{3,cc}}{T_h - T_c} \qquad G_{32} = \frac{N_{3,ch} - N_{3,cc}}{T_h - T_c}$$

Then if we define $Y_{ch} = N_{ch}/N_{cc}$, etc., we can get

$$T_e = \frac{T_h - Y_{hh} T_c}{Y_{hh} - 1}$$

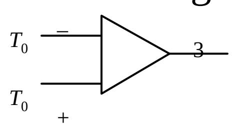
$$G_{31} + G_{32} = G_{3-} + G_{3+} = \frac{N_{hh} - N_{cc}}{T_h - T_c}$$

where we can use $Y_{hh} = Y_{ch} + Y_{hc} - 1$ (since we didn't measure hh).

- What about Noise Figure?
- Can define it as

$$F_3 = \left.\frac{total\ noise\ out}{noise\ out\ due\ to\ noise\ in}\right|_{T_0}$$
$$= 1 + \left.\frac{noise\ out\ due\ to\ amp}{noise\ out\ due\ to\ noise\ in}\right|_{T_0}$$
$$= 1 + \frac{(G_{31} + G_{32})T_e}{(G_{31} + G_{32})T_0}$$
$$= 1 + \frac{T_e}{T_0}$$

- Complication: this noise figure does *not* measure degradation of S/N.



$G_{3-}T_0 + G_{3+}T_0 + (G_{3-}+G_{3+})T_e$

$$F(S/N) = \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{N_{out}}{G_{3-}N_{in}}$$
$$= \frac{(G_{3-} + G_{3+})(T_0 + T_e)}{G_{3-}T_0}$$
$$= \left(1 + \frac{G_{3+}}{G_{3-}}\right)\left(1 + \frac{T_e}{T_0}\right) \approx \left(1 + \frac{T_e}{T_0}\right)$$

- Differs by factor of $(1+G_{3+}/G_{3-})$, due to difference in what is "input noise."

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